

A Singular Example in the Statistical Mechanics of a One-Dimensional System

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To examine boundary effects in phase transitions, the following simple one-dimensional model has been considered. Let particles characterized by a continuous spin $-1 \leq s_k \leq 1$ be located at the points $k = 0, 1, 2, \dots, N$ on the line subject to the nearest-neighbor interaction

$$\mathcal{H}_{k,k'} = J \ln 2\pi(1 - |s_k s_{k'}|) \quad (1)$$

In studying the partition function of the system as a function of the spin at $k = 0$, which is held fixed, one analytic result has been found which is presented here as of possible intrinsic interest.

In the thermodynamic limit the partition function is

$$Z(s_0) = \lim_{N \rightarrow \infty} 2^N \int_0^1 ds_1 \cdots \int_0^1 ds_N \exp\left\{-\beta J(\ln 2\pi)N - \beta J \sum_{k=1}^{\infty} \ln(1 - s_{k-1}s_k)\right\} \quad (2)$$

where β is the inverse temperature. This quantity satisfies the singular integral equation

$$Z(s) = 2e^{-\beta J \ln 2\pi} \int_0^1 dt \frac{Z(t)}{(1-st)^{\beta J}} \quad (3)$$

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An exact solution to this equation has been found for only one nontrivial case⁽¹⁾: $\beta J = 1$, where

$$Z(s_0) \sim 2^N(1 - |s_0|)^{-1/2} \mathbf{K}(|s_0|^{1/2}) \quad (4)$$

and \mathbf{K} denotes the complete elliptic integral of the first kind. Since $Z(s_0)$ is singular at $s_0 = \pm 1$ we are led to the conjecture that this system undergoes an ordering transition when the end spin is held fixed at ± 1 .

REFERENCE

1. M. L. Glasser, *SIAM Rev.* (to be published).